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Outsourcing optimization in two-echelon supply chain network under integrated production-maintenance constraints

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Abstract In this paper, we study a two-echelon supply chain network consisting of multi-outsourcers and multi-subcontractors. Each one is composed of a failure-prone production unit that produces a single product to fulfil market demands with variable production rates. Sometimes the manufacturing systems are not able to satisfy demand; in this case, outsourcing option is adopted to improve the limited in-house production capacity. The outsourcing is not justified by the production lack of manufacturing systems, but is also considered for the costs minimization issues. In the considered problem, we assume that the failure rate is dependent on the time and production rate. Preventive maintenance activities can be conducted to mitigate the deterioration effects, and minimal repairs are performed when unplanned failures occurs. We consider that the production cost depends on the rate of the machine utilization. The aim of this research is to propose a joint policy based on a mixed integer programming formulation to balance the trade-off between two-echelon of supply chain. We seek to assist outsourcers to determine the integrated in-house/ outsourcing, and maintenance plans, and the subcontractors to determine the integrated production-maintenance plans so that the benefit of the supply chain is maximized over a finite planning horizon. We develop an improved optimization procedure based on the genetic algorithms, and we discuss and conduct computational experiments to study the managerial insights for the developed framework.

Keywords Production-maintenance planning · In-house production · Outsourcing · Multiple costing schedule · Genetic algorithm · Outsourcing providers' selection · Failure-prone single machine

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Introduction and literature review

Outsourcing is a supply chain arrangement that allows an outsourcer to outsource some of its internal manufacturing processes to a subcontractor, or in some cases allows a subcontractor to outsource its resources to an outsourcer. This is, in order to cope with varied market demand or keep core competition. Outsourcing is commonly required as a tool to improve overall planning effectively and efficiently in different companies. Proper outsourcing can shorten lead times, reduce total costs, and make an organization more flexible (Lee and Choi 2011). Thus, the outsourcing that can provide a competitive advantage to companies is a completely new method that has emerged in recent years (Vaxevanou and Konstantopoulos 2015).

Integrated management approaches allow the management of several different areas within an organization. These approaches combine the requirements and objectives of the various functions so that the organization becomes more efficient to achieve the desired targets. It allows a global vision of the company by taking into account the interactions between different functions.

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It is claimed that subcontractors can provide to outsourcers a high quality with a lower price while using leading-edge technology. Most problems dealing with the outsourcing strategies consider outsourcing in production planning and do not take into account other parameters that could lead to additional costs for the company. Our motivations are directed towards the study of the mixed in-house production and outsourcing under integrated management approaches. However, subcontractors typically use their equipment with high rates to obtain more contracts, but without taking into account that the production unit will degrades quickly. To do this, subcontractors seek strategies to find a compromise between its objectives that consider the reliability and availability aspects of their production process and the outsourcer requirements in terms of price, delay, quantity and quality. For the outsourcers, the additional costs due to the exploitation of the unit with very high rates encourages the companies to outsource instead of using the in-house production. The problem is challenging in that, the study of outsourcing activities in integrated approach of production planning and maintenance management is nowadays a relevant problem in the field of supply chain optimization.

In addition, the outsourcing decisions depend on numerous factors whatever the level of decision. Some companies have achieved success with their outsourcing strategies, but others have experienced dismal failures. One cause for these failures is the lack of science-based decision methods and tools to help managers in making wise outsourcing decisions. This may further be due to relatively few studies on outsourcing strategies under integrated management approaches. This paper aims to provide an effective and efficient method for science-based decisions on the industrial outsourcing strategies under integrated approach of production control and maintenance planning problem.

There are many studies in the literature dealing with the association between different parties of the supply chain with the performance analysis of the outsourcing. For Thomas and Griffin (1996) and Gunasekaran et al. (2015), outsourcing is an operation strategy that improves the performances of supply chain. Some researchers proposed frameworks for outsourcing strategies. Abdel-Malek et al. (2005) proposed a framework to assess the performance of outsourcing strategies in a multi-layered supply chains. Gunasekaran et al. (2015) presented a classification of performance measures and metrics in strategic and tactical outsourcing engagements decisions. Bertrand and Sridharan (2001) studied a situation where demand rate is greater than the production rate, which implies the outsourcing of some orders. They considered that order lead times are exogenous and highly variable. Authors developed heuristics decision rules in order to minimize costs and maximize delivery reliability.

The issues related to the outsourcing in supply chains are very numerous. The most usual approach in the literature is to classify them according to the decision levels: for example, we find in the strategic level the problems of factories sitting or off-shoring and outsourcing providers' selection. In the tactical level, the balancing load-capacity (outsourcing, overtime...) are distinguished, and the production scheduling characterize the operational level (also called "flow control"). In this later context, Lee and Choi (2011) analyzed the computational complexity of a two-stage production scheduling problem. Two options for operations processing are considered: produce by utilizing in-house resources or outsource to a subcontractor. In Chen and Li (2008) and Tavares Neto et al. (2015) outsourcing is allowed in parallel to the associated in-house scheduling to promote the overall scheduling quality.

In the context of production planning, Kim (2003) considered a situation in which a company outsources its assembly operations to two different subcontractors; each one has owner level of improvement capability of inducing supply cost reduction. In Saharidis et al. (2009), authors treated a problem of production planning and compared global and local optimization for a two-stage supply chain with subcontracting options. Liu et al. (2008) developed a genetic algorithm heuristic method to solve a dynamic capacitated production planning problem with consideration of outsourcing. Authors consider that in-house manufacturing or outsourcing, without postponement or backlog, meets all demands. They also consider that levels of production, inventory, and outsourcing are limited. Lee and Lan (2013), study the problem of lot sizing with a secondary outsourcing facility. Authors considered the outsourcing to meet the unsatisfied part of the random demand. Following the same reasoning of Kim (2003), and Chiao et al. (2012) respond to the question: how a manufacturer determines the outsourcing quantity to be allocated to each outsourcer? Authors identifies two types of outsourcer: the first one offers a lower outsourcing price but has inferior facilities which result in a higher deteriorating rate, the other has advanced facilities causing a lower deteriorating rate but requires a higher outsourcing price. Zhen (2012) investigated a problem where an enterprise that manufactures multiple products in multiple periods. The author compares between two alternative modes: outsource parts or in-house manufacture parts and then assembles them in order to satisfy stochastic demands. He proposes an analytical approach to choice the optimal decision during each plan period.

In the outsourcing providers' selection point of view, Liou and Chuang (2010) proposed a hybrid multi-criteria decision-making model. However, in this work the question how orders are affected for each subcontractor is not addressed. Different models have been proposed, which tries to overcome such drawbacks. Wadhwa and Ravindran (2007) modeled this problem as a multi-objective optimization problem, where one or more outsourcers order multiple products from different subcontractors in a multiple sourcing network. The price, the lead-time and the rejects are considered as three criteria that have to be minimized simultaneously. Cui (2014) proposed an approach for jointly optimizing of production planning and supplier selection, considering customer flexibility. This problem has been formulated as a mixed integer programming model. The authors have developed a genetic algorithm in order to maximize the manufacturer's profit.

Other researchers considered outsourcing in the context of decision multi-levels. For instance, Boulaksil and Fransoo (2009) discussed and compared the performance of three different order release strategies to control outsourced operations between an original equipment manufacturer and a contract manufacturer that serves several clients.

In modern production systems, the components are usually reliable and preventive maintenance decisions should be integrated at the tactical level (Fitouhi and Nourelfath 2012). Several studies focusing on the joint planning of production and maintenance have been published. In Budai et al. (2008), a general overview of mathematical models that consider the interactions between production and maintenance is discussed. According to this study, the relation between production and maintenance exists in several ways. First, when planning maintenance takes production into account. Secondly, maintenance can also be seen as a production process which needs to be planned and finally, integrated models for production and maintenance. This integration has been studied in Boukas and Haurie (1990), Liao (2013), Njike et al. (2012), Wen et al. (2014). Weinstein and Chung (1999) examined the integration of maintenance and production decisions in hierarchical planning environment. Kenné and Gharbi (2004) proposed a stochastic optimization of production control problem with corrective and preventive maintenance. The objective was to determine the production and maintenance rates where the production and maintenance costs are minimized. Aghezzaf et al. (2007) have interested in a multi-item capacitated lot-sizing problems for a production system subject to random failures. The considered system is periodically renewed and minimally repaired at failure. Authors considered that maintenance action reduces the system's available production capacity.

The authors of previously mentioned studies discussed the value of combining maintenance and production planning. They showed that the integration of maintenance and production planning can reduce the total maintenance and production costs. Unlike of these works, those deals with cyclical preventive maintenance, other works have taken into account the possibility of noncyclical preventive maintenance. In Aghezzaf and Najid (2008) the same problem of Aghezzaf et al. (2007) is studied in parallel failure-prone production lines. The authors have proposed a Lagrangian-based heuristic procedure to solve this problem for both cyclical and noncyclical preventive maintenance policies. However, in Fitouhi and Nourelfath (2012) authors proposed a method to evaluate the capacity reduction, the times and the costs of preventive maintenance and minimal repair, and the average production system capacity. The model developed in our paper is different, and a method is proposed to evaluate the times and the costs of preventive maintenance and minimal repair. Our models determine simultaneously the optimal production and outsourcing plans and the instants of preventive maintenance actions.

The most of examined studies considered mainly the outsourcing in production scheduling and planning, lot sizing and production capacity reservation. While the outsourcing in those contexts has received much attention in the literature, but the outsourcing under a combined approach of production and maintenance have been less considered. In the literature, some research has been done on little close issues. For continuous models, Dellagi et al. (2010) have considered a manufacturing system subject to randomly failures, and do not able to respond to a constant demand. To fulfil this demand, the company call the outsourcing services. Authors have developed two strategies; the first one consists to choose one subcontractor among several. They have demonstrated analytically that the choice of subcontractors is conditioned by the unitary cost of lost sales. The second strategy consists to relay between two subcontractors. Dahane et al. (2011) have developed an integrated maintenance policy, which combines just-in-time production policy and maintenance decisions in a subcontracting environment. The authors considered a production system composed of two machines subjects to breakdowns and produce a single product type. The subcontracting is studied according to two perspectives: the subcontractors' orientation and the outsourcers' orientation.

Contrary in continuous time models, researchers have interested to discrete time models. Among them, we find the work of Hajej et al. (2014) that treated the problem of jointly production and maintenance considering subcontracting and product returns. The authors used a linear quadratic stochastic optimization problem for determining the optimal integrated plan. In these previous cited works, outsourcing is considered as operational constraint.

Our models are slightly different from the one developed by these studies. In their models, the authors interested in production and maintenance strategies. They developed integrated approaches of production-maintenance for units subject to outsourcing constraints. In contrast, we analyze the outsourcing optimization under integrated production-maintenance approach. The outsourcing optimization problem under combined approach of production and maintenance is justified by the fact that for the outsourcers, the additional cost of maintenance due to the exploitation of the machine with very high rates encourages the companies to outsource instead of using the in-house production. Similarly, subcontractors do not engage in a contract unless it is profitable. Therefore, the production at high rate accelerates the machine degradation and therefore increases the total cost of repairs, thus, “it is better not to outsource than to outsource without taking into account the machines degradation and additional costs”.

Some researchers interested to this view of point. For example, Dahane and Rezg (2011) have provided an economic model of outsourcing for a single-subcontractor multi-outsourcers relationship; the goal was to study the feasibility and the profitability of outsourcing in a joint maintenance/production context. Haoues et al. (2011) studied the optimization of outsourcing activities under a combined approach of maintenance/production. The authors proposed a global model for simultaneous optimization of the profits between single outsourcer and single subcontractor under a win-win partnership. Haoues et al. (2013), have proposed an integrated optimization of in-house production and outsourcing strategy under reliability constraints. The authors have mostly orientated on the alternation between in-house production and outsourcing strategies. The aspect of integrated maintenance is slightly integrated in this work. Recently, Rivera-Gómez et al. (2016) studied an unreliable deteriorating production system producing compliant and non-compliant products to fulfil a constant demand. The considered system is composed of a failure-prone single machine. Authors suppose that, due to the joint effect of random availability variations and deterioration, the manufacturing system is not capable to satisfy long-term part demand. In particular, when the stock level is positive, clients demand are satisfied just in time. When accumulation exists, outsourcing options are adopted at a higher cost to enhancement the limited in-house production capacity of the production system. The effect of deterioration is observed mainly in the quality of the products produced by in-house resources, since the rate of non- compliant parts increases as the machines deteriorate. Replacement activities can be performed to mitigate the deterioration effects. Authors propose a joint policy based on a stochastic dynamic programming formulation, which aims simultaneously to determine the production and maintenance rates, and the rate at which subcontractors are called. The proposed policy minimizes the total costs over an infinite planning horizon.

The present paper aim to help the managers with an integrated policy and decision-making tool. In reality, there are several types of outsourcing platform: platforms created by outsourcers, platforms created by subcontractors and platform for connections inter-company. We focus on the last type where an independent agent manages the relationship between the two parts of supply chain. First, all subcontractors adhere to the outsourcing platform which minimizes competition between them. Each subcontractor must meet its strategic clients and proposes a plan for secondary outsourcing, seeking to optimize partners' research. In the other hand, outsourcers propose projects for subcontracting and seek minimal costs. The platform receives the outsourcing requirements of both parties, analyses this requirements and returns the answer to each part.

The remainder of the paper is organized as follows: in the next section, the contributions of the paper are clearly presented. Section “Problem description” describes the considered system. The mathematical models have been developed in “Mathematical models formulation” while the computational procedure is discussed in “Genetic algorithm computational procedure”. Numerical study is carried out and some practical inferences are derived in “Experimental design and computational results? analysis”. Finally, conclusions and future scope of work are provided in “Conclusions and future research directions”.

Paper contributions

This research contributes to the small literature body on the association in two-echelon supply chain management with outsourcing strategies. More clearly, on outsourcing optimization under integrated production-maintenance approach. At this level, for the outsourcers, the decisions involve how to balance between in-house production, outsourcing, and maintenance. In another word, the determination of production rates of in-house, outsourcing, and maintenance strategies throughout sharing between the appropriate subcontractors. For the subcontractors, the decisions involve how to unhook additional contracts via the platform while optimizing partners' research.

Viewpoint reliability and maintenance, we consider the problem of integrating noncyclical preventive maintenance and tactical in-house production/outsourcing planning for a single machine. The maintenance policy suggests possible preventive replacements those depends on observed production rates, and minimal repair at machine failure. The proposed models determine simultaneously the optimal in-house production, outsourcing, secondary outsourcing plans and the instants of preventive maintenance actions. The objective is to maximize the net profits of outsourcers and subcontractors. Thus, minimize the sum of preventive and corrective maintenance costs, production costs, holding costs, outsourcing costs and shortage costs.

Most of the studies on supply chain management with outsourcing have considered mainly the subcontractor or outsourcer perspective, and optimize only one party. This does not necessarily lead to an optimal situation for all members of a supply chain (Jain et al. 2015; Behnamian and Fatemi Ghomi 2016). Even works that consider the two parts, most ones take into account that the subcontractor produces only for the outsourcer and does not have its own portfolio. For the capacity reservation problems, authors consider only the outsourcers parts, the subcontractors parts have considered as demands requirements. Some studies have no sufficient attention to outsourcing motivations, i.e. some consider the inability of production capacity, cost issue, and specialty. In our paper, we propose models with outsourcing motivations, including production capacity and cost issue.

The outsourcing problem considered in this paper has the following features: (1) part or all demand can be outsourced, all outsourcing requests are satisfied without postponement or backlogging, (2) each parts has costing schedule based on production or outsourced production rate, and (3) outsourcing level has a limit.

The basic objective of this research is to propose an integrated policy; we study how to balance the trade-off between outsourcers and subcontractors, and we aim to increase the efficiency of this system through outsourcing platform. The subcontractors without going through the platform should satisfy the strategic outsourcers. In addition, they seeks to unhook additional contracts via the platform. However, the outsourcers seek how to balance between in-house production and outsourcing, and propose outsourcing requests while seeking minimal costs. More clearly, we find the best integrated in-house production/outsourcing rates and maintenance plans for the outsourcers. The best integrated production rates and maintenance plans are found for the subcontractors.

Problem description

In this paper, we consider a two-echelon supply chain consisting of multi-outsourcers and multi-subcontractors. We model the production facility as a failure-prone single-machine that produces a single product, to satisfy the deterministic market demand with variable production rates during a given planning horizon H_r^a including N_r^a periods of exogenous length $\tau_{k_r}^a$, with $r = i, j$ (where i, respectively j, is the index of outsourcer, respectively subcontractor) and $a = \{o, s\}$, where o, respectively s, indicate that the actor is an outsourcer, respectively a subcontractor. $k_i = 1, \ldots, N_i^o$, and $k_j = 1, \ldots, N_j^s$.

Each demand d_{r,k_r}^a is to be satisfied at the end of the period. We consider that the production cost depends on the rate of the machine utilization. In addition, the failure rate is assumed dependent on the time and the production rate.. We consider an outsourcing platform, where subcontractors and outsourcers adhere from secondary or principal contracts, which maximize their profits and the profit of the supply chain. Each subcontractor (denoted s_j, $j = 1, \ldots, m$), has a relationship with strategic and secondary outsourcers. It should meet its strategic outsourcers without going through the platform, and seeks to unhook the additional contract (with secondary outsources) via the platform while optimizing partners' research. However, the secondary outsourcers (denoted o_i, $i = 1, \ldots, n$) seek how to balance the trade-off between in-house production and outsourcing, and propose outsourcing requests while seeking minimal costs. The platform receives the outsourcing requirements of both parts, analyses them and returns the answer to each part. Figure 1 shows the supply chain network structure considered in this paper.

Notations

Notations used throughout this paper are listed as follows:
Models Indices

a	Index of actor type of supply chain, $a = \{o, s\}$; where o, respectively s indicate that the actor is an outsourcer, respectively a subcontractor.
r	Index of actor of supply chain, with $r = \{i, j\}$; where i, respectively j is respectively the index of outsourcer and subcontractor.
k_r	Index of periods of actor r, $k_r = 1, \ldots, N_r^a$; where N_r^a is the number of orders of actor a.
l_r	Index of level in the cost model schedule, $l_r = 1, \ldots, L_r$; where L_r is the number of levels.
i	Index of outsourcer, where $i = 1, \ldots, n$; n is the number of outsourcers.
j	Index of subcontractor, where $j = 1, \ldots, m$; m is the number of subcontractors.

Decision variables

$u_{r,k_r,l_r}^{a,\overline{wo}}$ and $u_{r,k_r,l_r}^{a,wo}$	Production rate of outsourcer or subcontractor during period k_r, and its corresponding cost level l_r of production cost schedule (without, respectively with outsourcing option for outsourcer and without, respectively with secondary outsourcing for subcontractor).
$qin_{i,k_i,l_i}^{\overline{wo}}$ an qin_{i,k_i,l_i}^{wo}	In-house production quantity of outsourcer i, during period k_i produced with production rate $u_{i,k_i,l_i}^{o,\overline{wo}}$ and/or $u_{i,k_i,l_{i-1}}^{o,\overline{wo}}$ respectively $u_{i,k_i,l_i}^{o,wo}$ and/or $u_{i,k_i,l_{i-1}}^{o,wo}$, and its corresponding cost level respectively l_i and/or l_{i-1} of production cost schedule.
qot_{i,k_i}	Outsourcing quantity of outsourcer i during the period k_i.
qot_{j,k_i,k_j,l_j}	Outsourcing quantity produced by the subcontractor j during period k_j for the outsourcer i, during period

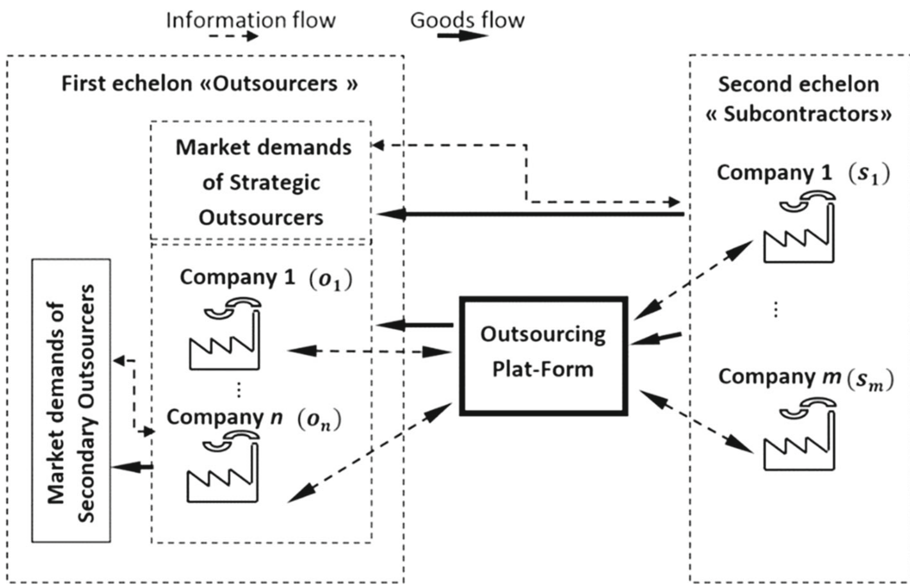

Fig. 1 Supply chain network considered in this study

	k_i, and its corresponding cost level respectively l_j and/or l_{j-1} of production cost schedule.
$T^a_{r,e}$	e^{th} optimal date of preventive maintenance of outsourcer or subcontractor.
e_{k_r}	Binary variable indicates that we change production rate or not during period k_r. It is given as follows: $e_{k_r} = \begin{cases} 1 \text{ if exist change in production rate} \\ 0 \text{ otherwise} \end{cases}$
$f_{MC}(T)$	Function of the expected total cost of maintenance strategy of the system per unit time T.
Npm^a_r	Number of PM during the planning horizon of outsourcer or subcontractor.
$\overline{sp}^o_i$	Average selling price of the outsourcer i.
$\overline{sp}^{s,st}_j$	Average selling price of the subcontractor j for the strategic outsourcers.
$\overline{sp}^{s,sc}_j$	Average selling price of the subcontractor j for the secondary outsourcers.
sc^o_i	Unit cost of shortage of outsourcer i.
I^a_{r,k_r}	Inventory level of the outsourcer i or subcontractor j at the end of period k_r.
$\pi^{a,\overline{wo}}_r$ and $\pi^{a,wo}_r$	Net profit of outsourcer or subcontractor (without, respectively with outsourcing option for outsourcer and without, respectively with secondary outsourcing for subcontractor).
π^{sc}_T	Total profit of supply chain.
$GR^{a,\overline{wo}}_r$ and $GR^{a,wo}_r$	Gross revenue of outsourcer or subcontractor (without, respectively with outsourcing option for outsocer and without, respectively with secondary outsourcing for subcontractor).
$PC^{a,\overline{wo}}_r$ and $PC^{a,wo}_r$	Total production cost of the outsourcer or subcontractor (without, respectively with outsourcing option for outsourcer and without, respectively with secondary outsourcing for subcontractor).
$IC^{a,\overline{wo}}_r$ and $IC^{a,wo}_r$	Total inventory holding cost of the outsourcer or subcontractor (without, respectively with outsourcing option for outsourcer and without, respectively with secondary outsourcing for subcontractor).
$MC^{a,\overline{wo}}_r$ and $MC^{a,wo}_r$	Total maintenance cost of the outsourcer or subcontractor (without, respectively with outsourcing option for outsourcer and without, respectively with secondary outsourcing for subcontractor).
$SC^{a,\overline{wo}}_r$ and $SC^{a,wo}_r$	Total shortage cost of the outsourcer or subcontractor (without, respectively with outsourcing option for outsourcer and without, respectively with secondary outsourcing for subcontractor).

OC_i^o Total outsourcing cost of the outsourcer i.

Models parameters

hc^a_{r,k_r} r^{th} outsourcer's or subcontractor's inventory holding cost per unit per unit time during period k_r.
pc^a_{r,k_r,l_r} Unit cost of production of outsourcer or subcontractor during the period k_r, and its corresponding cost level l_r of production cost schedule.
cmc^a_{r,k_r,l_r} Unit cost of corrective maintenance of outsourcer or subcontractor during the period k_r.
pmc^a_{r,k_r,l_r} Cost of a preventive maintenance intervention of outsourcer or subcontractor during the period k_r.
$\lambda^a_r(t)$ Hazard function of the machine of outsourcer or subcontractor at the instant t.
$umax^a_r$ Maximal production rate of outsourcer or subcontractor.
mg^a_r Margin rate or the percentage gain realized by the outsourcer or subcontractor r.
$\overline{te}$ Average time of execution.
$\alpha^a_r, \beta^a_{r,l_r}$ Parameters of the used failure distribution machine.

Assumptions

To describe the problem more clearly, we introduce the following assumptions:

- The planning horizon for each subcontractor or outsourcer is divided in Δy_r sub-periods indexed from $y = 1$ to $y = H^a_r$, where $\Delta y_r = 1$. These sub-periods are assumed not decomposable.
- Demands d^a_{r,k_r} arrive at the beginning of production plan horizon.
- The average demand of the strategic outsourcers should be relatively large compared to the demand of secondary outsourcers. In reality, the strategic outsourcers are actors whose subcontractors sign with them long durations and renewable contracts with large lots. However, the secondary outsourcers have occasional contracts with medium and small lots.
- The subcontractor should satisfy all demands of its strategic outsourcers.
- Subcontractors should deliver to outsourcers the demands or parts of the demands within the lead times specified by the latter. So, subcontractors assume the storage costs.
- The demand addressed to the outsourcer who is not satisfied will lost and generates a shortage cost.
- Production cost is variable and depends on the production rate.
- The processing time of order transactions is not considered.
- The inventory level considered is the final inventory at each sub-period.
- Inventory levels at both the beginning and the end of the planning horizon are not null.
- Failures are detected instantaneously.
- Durations of minimal repairs and preventive maintenance actions are negligible. The preventive maintenance actions should be performed at the end of the sub period.

Maintenance policy

To take into account the reliability and maintenance aspect, we suppose a single machine in each manufacturing system of considered supply chain those are subject to random failures, and the failure time for each machine, is governed by a Weibull probability distribution. The maintenance policy suggests preventive maintenance (PM) planning in order to reduce the increasing risk of machine failure. It is supposed that PM is perfectly performed, and each action restores the machine to "As Good As New" condition, or replaces the machine by a new one. Throughout this paper, sometimes we refer to such perfect PM as preventive replacement (PR). Furthermore, minimal repairs (MR) are carried out whenever an unplanned machine failure occurs, i.e., the machine is restored to an operating condition, but machine age is not reduced ("As Bad As Old" configuration). In practice, upon machine failure, the service team does just enough maintenance to make the machine usable.

We consider that the machine's hazard rate $\lambda^a_{k_r}\left(t, u^a_{r,k_r,l_r}\right)$ increases with the time and depends on the production rate. So, the production at high rate accelerates the machine degradation and therefore increases the risk of failure, consequently, the number and the total cost of minimal repairs increase (Haoues et al. 2011).

For a given period $k_r = [\tau_{k_r-1}, \tau_{k_r}], k_r \geq 1$ the hazard rate is written as follows:

$$\lambda^a_{k_r}(t) = \frac{u^a_{r,k_r,l_r}}{umax^a_r}\,{}_{\lambda}max^a_r(t) \tag{1}$$

were λmax^a_r is the nominal hazard rate corresponding to the maximal production rate of outsourcer or subcontractor.

Since we assume repair is minimal, we can model the occurrence of failures during each period of planning horizon k_r using a Non-Homogeneous Poisson Process. Then, the expected number of failures is given by:

$$M\left(H^a_r\right) = \sum\nolimits_{k_r=1}^{N^a_r} \int_{\tau_{k_r-1}}^{\tau_{k_r}} \lambda^a_{k_r}(t)\,dt \tag{2}$$

Since we assume PM restores the machine to AGAN condition, we can model the operation and maintenance of the machine as a Renewal Process, where the renewal points are: the initiation of machine operation and the end of each PM activity (Cassady and Kutanoglu 2005). We perform PM at the optimal date $T^a_{r,e}$; the effective age of the machine becomes zero, because the machine is renewed. This implies that PM is a more comprehensive action than repair, perhaps corresponding to the replacement of several key components in the machine.

The model developed in this paper takes into account the MR and noncyclical PM; this later depends on observed production rate during the periods of plan.

In-house production/outsourcing policies

The subcontractor's production control policy consists in (1) building a stock of finished products, having a production capacity greater than or equal to the requirements of strategic outsourcers, to satisfy these latter and (2) produce for the secondary outsourcers by offering secondary outsourcing services; these are based on orders from the outsourcers and contractual obligations.

For the secondary outsourcers, the in-house production capacity is, sometimes less than the global market demand. In this case, in order to satisfy very high demands beyond predictions, without postponement or backlogging, the company has to resort to the outsourcing to fill in the gap between in-house production and demands. However, the use of outsourcing is not always justified by the inability of in-house production, but also due to the consideration of costs minimization issue. Based on the principle of comparative advantage, the secondary outsourcer chooses the in-house production plan that minimizes the total costs (Zhen 2012). Results demonstrate that outsourcer's high in-house costs motivate firms to outsource to an independent subcontractor (Ni et al. 2009). Nowadays, in the global supply chain network, few firms manufacture all the parts of a product products by in-house manufacturing without outsourcing.

We suppose that machines in each part of supply chain operate with variable production rates for each production period. We consider dependence between rate and cost of production, this one is justified by the fact that in economics, production cost consists of two distinct components: fixed costs and variable costs. Fixed costs are expenses that are not dependent on the level of goods or services produced by the enterprise. They tend to be time-related, such as labor hand, electricity or rents, and are often referred to as overhead costs. This is in contrast to variable costs, which are volume-related, and are paid per quantity produced.

Thus, operating costs of the machine depends on their production rate. Let pc^a_{r,k_r,l_r} be the cost model schedule with L_r levels $\left\{pc^a_{r,k_r,l_r}, u^a_{r,k_r,l_r}\right\}$, $l_r = 1, 2, \ldots, L_r$. It consists of the production cost pc^a_{r,k_r,l_r} and the corresponding common/group production rate u^a_{r,k_r,l_r}. The following formula is the general form of the cost schedule:

$$pc^a_{r,k_r,l_r} = \begin{cases} pc^a_{r,k_r,1} & \text{if } umin^a_r < u^a_{r,k_r,l_r} \le u^a_{r,k_r,1} \\ \ldots & \ldots \\ pc^a_{r,k_r,l_r} & \text{if } u^a_{r,k_r,l_r-1} < u^a_{r,k_r,l_r} \le u^a_{r,k_r,l_r} \\ \ldots & \ldots \\ pc^a_{r,k_r,L_r} & \text{if } u^a_{r,k_r,L_r} < u^a_{r,k_r,l_r} \le umax^a_r \end{cases} \tag{3}$$

where $pc^a_{r,k_r,1} < \ldots < pc^a_{r,k_r,l_r} < \ldots < pc^a_{r,k_r,L_r}$ and $umin^a_r < u^a_{r,k_r,1} < \ldots < u^a_{r,k_r,l_r-1} < u^a_{r,k_r,l_r} < \ldots < u^a_{r,k_r,L_r} < umax^a_r$

Suppose we use the machine with production rates $u^a_{r,k_r,1}, \ldots, u^a_{r,k_r,l_r}, \ldots, \quad u^a_{r,k_r,L_r}$.

According to the dependence relationship between failure rate and production rate we have:

$$\lambda^a_{r,1}(t) < \ldots < \lambda^a_{r,l_r}(t) < \ldots < \lambda^a_{r,L_r}(t) \tag{4}$$

i.e. if we use the machine with a production rate u_r the machine failure rate is $\lambda_{l_r}(t)$. Consequently, we have the following expression of the maintenance costs:

$$MC^a_{k_r} = CMC^a_{k_r} + PMC^a_{k_r} \tag{5}$$

Therefore, $MC^a_{k_r,1} < \ldots < MC^a_{k_r,l_r} < \ldots < MC^a_{k_r,L_r}$ (6)

Thus, the in-house production rates will affect the total cost.

Mathematical models formulation

For the sake of brevity, we do not present the separated optimization models, and we develop only the integrated optimization. It is clear that each company uses its own mechanisms for determining the selling price. Generally, if a product is manufactured entirely, the average selling price for the outsourcers is calculated as follows:

$$\overline{sp^o_i} = \frac{PC^{o,\overline{wo}}_i + IC^{o,\overline{wo}}_i + MC^{o,\overline{wo}}_i}{\sum_{k_i=1}^{N^o_i} G^{o,\overline{wo}}_{i,k_i}}\left(1 + mg^o_i\right) \tag{7}$$

$$\text{With } G^{o,\overline{wo}}_{i,k_i} = \begin{cases} \left(Tmin^o_{i,k_i} - \tau_{k_i-1}\right) u^{o,\overline{wo}}_{i,k_i-1,l_{i-1}} \\ +\left(\tau_{k_i} - Tmin^o_{i,k_i}\right) u^{o,\overline{wo}}_{i,k_i,l_i} & \text{if } \exists T^o_{i,e} \in \left[\tau_{k_i-1}, \tau_{k_i}\right] \\ \Delta\tau_{k_i} u^{o,\overline{wo}}_{i,k_i,l_i} & \text{otherwise} \end{cases}$$

where, $u^{o,\overline{wo}}_{i,k_i,l_i}$ is the in-house production rate of outsourcer i, during subperiod $\left[T^o_{i,e-1}, T^o_{i,e}\right]$ and its corresponding cost level l_i of production cost schedule.

were $Tmin^o_{i,k_i} = \min\limits_{k_i} T^o_{i,k_i}$

Such as $T^o_{i,k_i} = \left\{T^o_{i,1}, \ldots, T^o_{i,e}, \ldots, T^o_{i,E}\right\}$, E is the number of preventive maintenance actions during period k_i.

The margin rate means the percentage gain realized by the company knowing that the margin rate includes the value-added tax amount. The cost price includes the sum of purchase costs excluding taxes of raw items and products used in the composition of the product for sale, labor, social contribution, depreciation of the industrial tool and marketing costs. The average selling price for the subcontractors is expressed by the formula (8):

$$\overline{sp}^{s,st}_j = \frac{PC^{s,\overline{wo}}_j + IC^{s,\overline{wo}}_j + MC^{s,\overline{wo}}_j}{\sum_{k_j=1}^{N^s_j} d^s_{j,k_j}} \left(1 + mg^s_j\right) \tag{8}$$

In separate approach, the optimization criterion is the maximization of the profit of each actor of supply chain separately.

Outsourcer's integrated planning model with outsourcing

The outsourcers can use outsourcing as a secondary supplying source in addition to the in-house production, to handle demand fluctuations without the need of maintaining a high production or inventory capacity (Qi 2011). However, the use of outsourcing is not always justified by the inability of in-house production, but also due to the consideration of costs minimization issue. The outsourcing tasks requires a new optimization of the production-maintenance plan based on a new production rates. This formulation allows answering to the following question: how to balance the trade-off between the two modes: in-house production and outsourcing, i.e. choose between outsource, in-house production or assume shortage cost. The expression of the outsourcer net profit with the outsourcing option is given as follows:

$$\pi^{o,wo}_i = GR^{o,wo}_i - \left(PC^{o,wo}_i + IC^{o,wo}_i + SC^{o,wo}_i + OC^{o,wo}_i + MC^{o,wo}_i\right) \tag{9}$$

Gross revenue

Gross revenue is the amount of overall sales performed during the planning horizon; it includes all sales, whether the part demand produced by in-house resources or the outsourced part. It is given by the formula (10):

$$GR^{o,wo}_i = \begin{cases} \overline{sp}^o_i \left(\sum_{k_i=1}^{N^o_i} qin^{wo}_{i,k_i,l_i} + \sum_{k_i=1}^{N^o_i} \sum_{j=1}^{m} \sum_{k_j=1}^{N^s_j} qot_{j,k_i,k_j,l_j} \right) & \text{if } i=1 \\ \overline{sp}^o_i \left(\sum_{k_i=1}^{N^o_i} qin^{wo}_{i,k_i,l_i} + \sum_{k_i=T+1}^{T+N^o_i} \sum_{j=1}^{m} \sum_{k_j=1}^{N^s_j} qot_{j,k_i,k_j,l_j} \right) & \text{otherwise} \end{cases} \tag{10}$$

Such as, $T = \sum_{t=1}^{i-1} N^o_t$

where, qin_{k_i,l_i} is the in-house production quantity of outsourcer i, during period k_i produced with production rate $u^{o,wo}_{i,k_i,l_i}$ and/or $u^{o,wo}_{i,k_i-1,l_{i-1}}$, and its corresponding cost level respectively l_i and/or l_{i-1} of production cost schedule. It is expressed by the following formula:

$$qin_{k_i,l_i} = \begin{cases} \left(Tmin^o_{i,k_i} - \tau_{k_i-1}\right) u^{o,wo}_{i,k_i-1,l_{i-1}} + \left(\tau_{k_i} - Tmin^o_{i,k_i}\right) u^{o,wo}_{i,k_i,l_i} & \text{if } \exists\ T^o_{i,e} \in \left[\tau_{k_i-1}, \tau_{k_i}\right] \\ \Delta\tau_{k_i} u^{o,wo}_{i,k_i,l_i} & \text{otherwise} \end{cases} \tag{11}$$

Inventory holding cost

The inventory level considered is the final inventory at each sub-period. The storage of the product generates holding costs, the total inventory holding cost is given by the following expression:

$$IC^{o,wo}_i = \sum_{k_i=1}^{N^o_i} hc^o_{i,k_i} \left(\left\lfloor I_{k_i-1} \right\rfloor \Delta\tau_{k_i} + \sum_{y=2}^{\tau_{k_i}} (y-1) \left\lfloor u^{o,wo}_{i,k_i,l_{y-1}} + I_{y-1} - \left\lfloor I_{y-1} \right\rfloor \right\rfloor \right) \tag{12}$$

The Fig. 2 presents an example of production–inventory graph evolution of outsourcer. It shows how we changes and adopts the production rates, the production rates depends on the generated chromosomes. Were:

$$u_{k_i+1} = \begin{cases} u'_{k_i} & \text{if } \exists T^a_{i,e} \in \left[\tau_{k_i-1}, \tau_{k_i}\right] \\ u_{k_i} & \text{otherwise} \end{cases}$$

For determine the level of stock at the end of each period, we distinguish two cases. The first one is when if $\exists\ T^o_{i,e} \in \left[\tau_{k_i-1}, \tau_{k_i}\right]$:

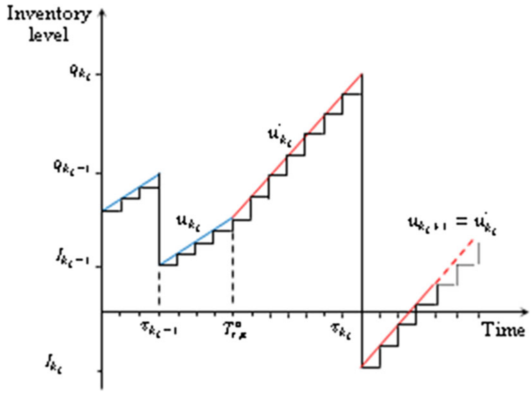

Fig. 2 A production—inventory graph evolution of outsourcer

Shortage cost

The shortage cost generated by outsourcer with outsourcing option is expressed by the following formula:

$$SC_i^{o,wo} = sc_i^o \sum\nolimits_{k_i=1}^{N_i^o} SL_{i,k_i}^{o,wo} \tag{15}$$

where, $SL_{i,k_i}^{o,wo}$ is the shortage level (shortage quantity) at the end of each period. We distinguish two cases. The first one is when if $\exists T_{i,e}^o \in \left[\tau_{k_i-1}, \tau_{k_i}\right]$:

$$I_{k_i} = \begin{cases} \max\left[0, \left(\begin{array}{l} I_{k_i-1} + \left(Tmin_{i,k_i}^o - \tau_{k_i-1}\right) u_{i,k_i-1,l_{i-1}}^{o,wo} + \left(\tau_{k_i} - Tmin_{i,k_i}^o\right) u_{i,k_i,l_i}^{o,wo} \\ + \sum\limits_{k_i=1}^{N_i^o} \sum\limits_{j=1}^{m} \sum\limits_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j} \end{array}\right) - d_{i,k_i}^o\right] & \text{if } i = 1 \\ \max\left[0, \left(\begin{array}{l} I_{k_i-1} + \left(Tmin_{i,k_i}^o - \tau_{k_i-1}\right) u_{i,k_i-1,l_{i-1}}^{o,wo} + \left(\tau_{k_i} - Tmin_{i,k_i}^o\right) u_{i,k_i,l_i}^{o,wo} \\ + \sum\limits_{k_i=T+1}^{T+N_i^o} \sum\limits_{j=1}^{m} \sum\limits_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j} \end{array}\right) - d_{i,k_i}^o\right] & \text{otherwise} \end{cases} \tag{13}$$

The second case is when if $\nexists\, T_{i,e}^o \in \left[\tau_{k_i-1}, \tau_{k_i}\right]$:

$$I_{k_i} = \begin{cases} \max\left[0, \left(I_{k_i-1} + \Delta\tau_{k_i} u_{i,k_i,l_i}^{o,wo} + \sum\limits_{k_i=1}^{N_i^o} \sum\limits_{j=1}^{m} \sum\limits_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j}\right) - d_{i,k_i}^o\right] & \text{if } i = 1 \\ \max\left[0, \left(I_{k_i-1} + \Delta\tau_{k_i} u_{i,k_i,l_i}^{o,wo} + \sum\limits_{k_i=T+1}^{T+N_i^o} \sum\limits_{j=1}^{m} \sum\limits_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j}\right) - d_{i,k_i}^o\right] & \text{otherwise} \end{cases} \tag{14}$$

$$SL_{i,k_i}^o = \begin{cases} \max\left[0, d_{i,k_i}^o - \left(\begin{array}{l} I_{k_i-1} + \left(Tmin_{i,k_i}^o - \tau_{k_i-1}\right) u_{i,k_i-1,l_{i-1}}^{o,wo} + \left(\tau_{k_i} - Tmin_{i,k_i}^o\right) u_{i,k_i,l_i}^{o,wo} \\ + \sum\limits_{k_i=1}^{N_i^o} \sum\limits_{j=1}^{m} \sum\limits_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j} \end{array}\right)\right] & \text{if } i = 1 \\ \max\left[0, d_{i,k_i}^o - \left(\begin{array}{l} I_{k_i-1} + \left(Tmin_{i,k_i}^o - \tau_{k_i-1}\right) u_{i,k_i-1,l_{i-1}}^{o,wo} + \left(\tau_{k_i} - Tmin_{i,k_i}^o\right) u_{i,k_i,l_i}^{o,wo} \\ + \sum\limits_{k_i=T+1}^{T+N_i^o} \sum\limits_{j=1}^{m} \sum\limits_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j} \end{array}\right)\right] & \text{otherwise} \end{cases} \tag{16}$$

The second case is when if $\nexists\, T^{o*}_{i,e} \in \left[\tau_{k_i-1}, \tau_{k_i}\right]$:

$$SL^{o}_{i,k_i} = \begin{cases} \max\left[0, d^{o}_{i,k_i} - \left(I_{k_i-1} + \Delta\tau_{k_i} u^{o,wo}_{i,k_i,l_i} + \sum_{k_i=1}^{N_i^o}\sum_{j=1}^{m}\sum_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j}\right)\right] & \text{if } i = 1 \\ \max\left[0, d^{o}_{i,k_i} - \left(I_{k_i-1} + \Delta\tau_{k_i} u^{o,wo}_{i,k_i,l_i} + \sum_{k_i=T+1}^{T+N_i^o}\sum_{j=1}^{m}\sum_{k_j=1}^{N_j^s} qot_{j,k_i,k_j,l_j}\right)\right] & \text{otherwise} \end{cases} \tag{17}$$

Outsourcing cost

The outsourcer is looking to outsource with a minimal cost, however a strong relation exist between outsourcing costs and demand quantity and its delivering time. This interdependence was detailed and explained in "Problem description". The total outsourcing cost can be written as follows:

$$OC^{o,wo}_{i} = \begin{cases} \sum_{k_i=1}^{N_i^o}\sum_{j=1}^{m}\sum_{k_j=1}^{N_j^s} \overline{sp}^{s,sc}_{j} qot_{j,k_i,k_j,l_j} & \text{if } i = 1 \\ \sum_{k_i=T+1}^{T+N_i^o}\sum_{j=1}^{m}\sum_{k_j=1}^{N_j^s} \overline{sp}^{s,sc}_{j} qot_{j,k_i,k_j,l_j} & \text{otherwise} \end{cases} \tag{18}$$

Such as, $\boldsymbol{T = \sum_{t=1}^{i-1} N_t^o}$

Production cost

The total production cost of each actor of supply chain (outsourcer or subcontractor) is given by the following expression:

$$PC^{a,wo}_{r} = \sum_{k_r=1}^{N_j^a} PC^{a,wo}_{r,k_r} \tag{19}$$

where, the production cost by period is written as follows:

$$PC^{a,wo}_{r,k_r} = \begin{cases} \begin{pmatrix} \left(Tmin^{a}_{r,k_r} - \tau_{k_r-1}\right) u^{a,wo}_{r,k_r-1,l_{r-1}} pc^{a}_{r,k_r,l_{r-1}} + \\ \left(\tau_{k_r} - Tmin^{a}_{r,k_r}\right) u^{a,wo}_{r,k_r,l_r} pc^{a}_{r,k_r,l_r} \end{pmatrix} & \text{if } \exists\, T^{a}_{r,e} \in \left[\tau_{k_r-1}, \tau_{k_r}\right] \\ pc^{a}_{r,k_r,l_r} . \Delta\tau_{k_r} . u^{a,wo}_{r,k_r,l_r} & \text{otherwise} \end{cases} \tag{20}$$

Maintenance cost

The expected maintenance cost of each actor of supply chain is given by the following expression:

$$MC^{a,wo}_{r} = CMC^{a}_{r} + PMC^{a}_{r} \tag{21}$$

where the total preventive maintenance cost is given by the following expression:

$$PMC^{a}_{r} = \sum\nolimits_{k_r=1}^{N_r^a} e_{k_r}\, pmc^{a}_{r,k_r-1,l_{r-1}} + \left(\mathrm{E} - e_{k_r}\right) pmc^{a}_{r,k_r,l_r} \tag{22}$$

The total corrective maintenance cost is given as follows:

$$CMC^{a}_{r} = \sum\nolimits_{k_r=1}^{N_r^a} CMC^{a}_{k_r} \tag{23}$$

The corrective maintenance cost per period is given by the formula (24):

$$CMC^{a}_{r} = \begin{cases} \begin{pmatrix} cmc^{a}_{r,k_r-1,l_{r-1}} \frac{u^{a,wo}_{r,k_r-1,l_{r-1}}}{umax^{a}_{r}} \int_{\tau_{k_j-1}}^{Tmin^{a}_{r,k_r}} \lambda max^{a}_{r}(t)\, dt \\ + cmc^{a}_{r,k_r,l_r} \frac{u^{a,wo}_{r,k_r,l_r}}{umax^{a}_{r}} \int_{Tmin^{a}_{r,k_r}}^{\tau_{k_r}} \lambda max^{a}_{r}(t)\, dt \end{pmatrix} & \text{if } \exists\, T^{a}_{r,e} \in \left[\tau_{k_r-1}, \tau_{k_r}\right] \\ cmc^{a}_{r,k_r,l_r} \frac{u^{a,wo}_{r,k_r,l_r}}{umax^{a}_{r}} \int_{\tau_{k_r-1}}^{\tau_{k_r}} \lambda max^{a}_{r}(t)\, dt & \text{otherwise} \end{cases} \tag{24}$$

Subcontractor's integrated planning model with secondary outsourcing

For the subcontractors, the outsourcing is a way to increase the exploitation of the machine capacity and the generated profits. The additional tasks of outsourcing, requires a new optimization of the production-maintenance plan based on a new production rates. The selling price for strategic outsourcers was determined in the separate approach and we determine the selling price for secondary outsourcers. The expression of the subcontractor net profit with additional outsourcing option is given as follows:

$$\pi_j^{s,wo} = GR_j^{s,wo} - \left(PC_j^{s,wo} + IC_j^{s,wo} + MC_j^{s,wo}\right) \tag{25}$$

Gross revenue

Gross revenue includes all sales, whether the part demand produced for the strategic or the secondary outsourcers. It is given by the formula (26):

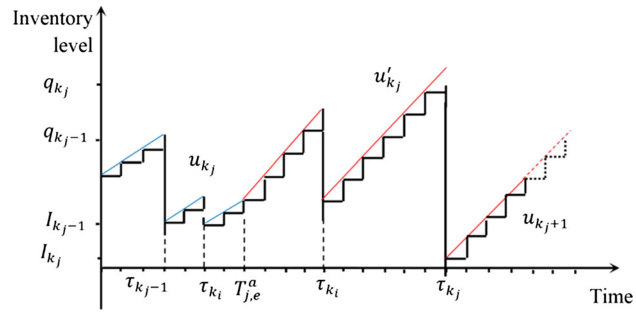

Fig. 3 A production—inventory graph evolution of subcontractors

$$GR_j^{s,wo} = \begin{cases} \overline{sp}_j^{s,st} \sum_{k_j=1}^{N_{s_j}} d_{j,k_j}^s + \sum_{k_i=1}^{N_i^o} \sum_{j=1}^{m} \sum_{k_j=1}^{N_j^s} \overline{sp}_j^{s,sc} qot_{j,k_i,k_j,l_j} & \text{if } i = 1 \\ \overline{sp}_j^{s,st} \sum_{k_j=1}^{N_{s_j}} d_{j,k_j}^s + \sum_{k_i=T+1}^{T+N_i^o} \sum_{j=1}^{m} \sum_{k_j=1}^{N_j^s} \overline{sp}_j^{s,sc} qot_{j,k_i,k_j,l_j} & \text{otherwise} \end{cases} \tag{26}$$

Inventory holding cost

As for the outsourcer, the Fig. 3 presents an example of production–inventory graph evolution to shows how calculate the inventory level of each period. It is given by the expression (27):

$$\begin{aligned} IC_j^{s,wo} = \sum_{k_j=1}^{N_j^s} hc_{j,k_i}^s \Big(& \lfloor I_{k_j-1} \rfloor \left(\tau_{1-}^o \tau_{k_{j-1}}\right) \\ & + \sum_{y=2}^{\tau_{1-}^o \tau_{k_{j-1}}} (y-1) \left\lfloor u_{j,k_j,l_{y-1}}^{o,wo} + I_{y-1} - \lfloor I_{y-1} \rfloor \right\rfloor + \cdots \\ & + \left\lfloor I_{\tau_{n-1}^o} \right\rfloor \left(\tau_{n-}^o \tau_{n-1}^o\right) + \sum_{y=2}^{\tau_{n-}^o \tau_{n-1}^o} (y-1) \left\lfloor u_{j,k_j,l_{y-1}}^{o,wo} + I_{y-1} - \lfloor I_{y-1} \rfloor \right\rfloor \\ & + \cdots \left\lfloor I_{\tau_n^o} \right\rfloor \left(\tau_{k_j} - \tau_n^o\right) \sum_{y=2}^{\tau_{k_j} - \tau_n^o} (y-1) \left\lfloor u_{j,k_j,l_{y-1}}^{o,wo} + I_{y-1} - \lfloor I_{y-1} \rfloor \right\rfloor \Big) \end{aligned} \tag{27}$$

where $\zeta_j = \left[\tau_1^o, \cdots, \tau_i^o, \cdots, \tau_n^o\right]$ is the vector of delivering time of outsourcers ($i = 1, \cdots, n$) those serve by the subcontractor j. Such as $qot_{i,k_i} \neq 0$ in period k_j. The level of stock at the end of each period is written as follows:

$$I_{k_j} = \begin{cases} \max\left[0, \begin{pmatrix} I_{k_j-1} + \left(Tmin_{j,k_j}^s - \tau_{k_j-1}\right) u_{j,k_j-1,l_{j-1}}^{s,wo} + \\ \left(\tau_{k_j} - Tmin_{j,k_j}^s\right) u_{j,k_j,l_j}^{s,wo} \end{pmatrix} - d_{j,k_j}^s\right] & \text{if } \exists\, T_{j,e}^s \in \left[\tau_{k_j-1}, \tau_{k_j}\right] \\ \max\left[0, \left(I_{k_j-1} + \Delta\tau_{k_j} . u_{j,k_j,l_j}^{s,wo}\right) - d_{j,k_j}^s\right] & \text{otherwise} \end{cases} \tag{28}$$

Integrated optimization

In integrated approach, the optimization criterion is the maximization of the profit generated by the supply chain. The requests sending by the secondary outsourcers are sharing between the subcontractors, by respecting the minimal selling price of subcontractors, the delivering date imposed by the outsourcers and to find the best outsourcing quantities. In the case where several requests are instantaneously arrived, we ensure that a distribution proportional to the outsourced quantity requested is respected "fair allocation". The resulting mathematical programming formulation is given as follows:

The fitness function over the planning horizon is defined as:

$$\max\left(\sum_{i=1}^{n}\pi_i^{o,wo}+\sum_{j=1}^{m}\pi_j^{s,wo}\right)\ \forall i\in\{1,\ldots,n\};\ \forall j\in\{1,\ldots,m\} \tag{29}$$

Subject to equalities (12), (13) and (28) and

$$\pi_i^{o,wo}\geq\pi_i^{o,\overline{wo}}\quad\forall i\in\{1,\ldots,n\} \tag{30}$$

$$\overline{sp}_j^{s,sc}\leq sc_i^o\quad\forall i\in\{1,\ldots,n\};\forall j\in\{1,\ldots,m\} \tag{31}$$

$$f_{MC}=\frac{CMC_r^a+PMC_r^a}{T}\quad\forall r\in\{i,j\};\quad\forall a\in\{o,s\} \tag{32}$$

$$T^*/\frac{\partial f_{MC}(T)}{\partial T}=0 \tag{33}$$

$$0\leq u_{i,k_i,l_i}^{o,wo}\leq umax_i^o\qquad\forall i\in\{1,\ldots,n\} \tag{34}$$

$$\max\left(\frac{d_{j,k_j}^s-I_{k_j-1}}{\Delta\tau_{k_j}},0\right)\leq u_{j,k_j,l_j}^{s,wo}\leq umax_j^s\quad\forall j\in\{1,\ldots,m\} \tag{35}$$

$$qin_{i,k_i,l_i}^{wo},qot_{i,k_i},qot_{j,k_i,k_j,l_j},u_{i,k_i,l_i}^{o,wo},u_{j,k_j,l_j}^{s,wo},T_{r,e}^a,Npm_r^a,\overline{sp}_i^o,\overline{sp}_j^{s,st},\overline{sp}_j^{s,sc},sc_i^o,I_0\geq 0\quad\forall i\in\{1,\ldots,n\};\forall j\in\{1,\ldots,m\}\quad\forall r\in\{i,j\};\quad\forall a\in\{o,s\} \tag{36}$$

$$qin_{i,k_i,l_i}^{wo},u_{i,k_i,l_i}^{o,wo},u_{j,k_j,l_j}^{s,wo},\overline{sp}_i^o,\overline{sp}_j^{s,st},\overline{sp}_j^{s,sc},sc_i^o\in\mathbb{R}_+ \tag{37}$$

$$qot_{i,k_i},qot_{j,k_i,k_j,l_j},T_{r,e}^a,e_{k_i},Npm_r^a,I_{k_r}\in\mathbb{Z}_+\quad k_i=1,\ldots,N_i^o,\quad k_j=1,\ldots,N_j^s,\quad l_i=1,\ldots,L_i\ \text{and}\ l_j=1,\ldots,L_j \tag{38}$$

In the above, expression (29) represents the maximum of the total profit aggregated from each actor of supply chain in integrated optimization. The inequality constraint (30) ensures that the profit generated by each outsourcer i with outsourcing option is greater than or equal than the profit generated by this outsourcer without outsourcing. The constraint (31) ensures that each outsourcer i does not accept terms of outsourcing contract with subcontractor j, except if the selling price proposed by this later is smaller or equal than the shortage cost generated if the demand is not meet by the outsourcer. The equalities constraints (12), (13) and (28) shows that the inventory level at the end of each period k equals to stock aggregated from remaining stock of previous production period $k-1$, in-house production and outsourcing minus the demand for this period. The maintenance constraints are expressed through equalities (32) and (33). The former represents the expected total cost of the maintenance strategy of the machine while the later aims at determining the optimal preventive maintenance dates. The maintenance policy accounts for the MR and noncyclical PM; this later depends on observed production rate during the periods of plan. Under the assumption that the machine's hazard rate increases with both time and the production rate, the production at high rate accelerates the machine degradation. This in turn leads to an increase of the failure risk, consequently, the number and the total cost of minimal repairs augments. The outsourcers' machines can operate with production rates those depend on outsourcing options. The production rates of outsourcers are required within the lower and upper bounds by constraints (34). The constraints (35) show that the subcontractors can use their machines with a production rates included between minimal and maximal production rate. The constraints (36) represent non-negativity constraints. Finally, the constraints (37) and (38) provide the type of decision variables.

Genetic algorithm computational procedure

Motivation for a genetic algorithm approach

The problem of (Cui 2014) is slightly similar to our. Cui (2014) has treated the problem of jointly optimizing of production planning and supplier selection, considering customer flexibility. He considered a single manufacturer that produces multiple products. However, author does not consider the maintenance aspect, and the problem has been formulated as a mixed integer programming model.

In our problem, the mathematical models are formulated as a mixed integer programming model to assist multiple outsourcers in determining the mixed in-house and outsourcing plans, and assist multiple subcontractors in determining production plan taking into account the constraint of reliability and maintenance and selecting partners. As mentioned by Nakagawa and Mizutani (2009) it is impossible to determine analytically the PM instants for finite time horizons with fixed time to failure distribution. In our problem, the reliability and maintenance aspect is considered with a dynamic failure law, i.e. its parameters depend on the adopted production rate, also the determination of instants of PM is dependent on production rates observed in previous periods. Some constraints of the problem are nonlinear constrains; the complexity also increases when the number of actors in the supply chain, their periods and levels of costing schedule increases. Therefore, it is impossible to solve the above mentioned models by using exact optimization methods. In such situation, the

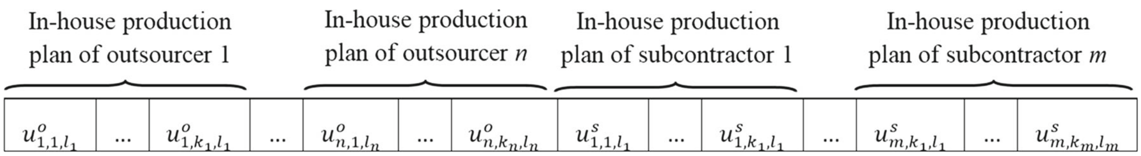

Fig. 4 Chromosome structure

evolutionary algorithms have been widely used by many researchers as an effective mean to trade-off between the global optimum and the computational complexity (Dellaert et al. 2000; Xie and Dong 2012). Accordingly, we have applied GA evolutionary computation techniques to solve efficiently the proposed models. These algorithms are founds to be more efficient in generating near global optimal solution for complex problems with less computational time; it has been successfully applied to many combinatorial optimization problems (Liu et al. 2008). Genetic algorithms have been recognized as a powerful and widely applicable optimization method, especially for solving global optimization problems and NP-hard problems. Accordingly, this study proposes a GA approach, as discussed in the following subsections.

Genetic algorithm meta-heuristic approach

To develop a GA computational procedure, we should have six elements. We explained these elements that complete a meaningful GA approach; they are the encoding scheme of the population element, a mechanism for generation of the initial population, a fitness function, genetic operators, stopping criteria and sizing parameters.

Encoding scheme of the population element

From the above proposed models in "Mathematical models formulation", we can see that all decision variables are dependent on the variables u^a_{r,k_r,l_r}. Therefore, we only encode the u^a_{r,k_r,l_r} as chromosomes.

i.e. $\left\{u^o_{1,1,l_1}, \ldots, u^o_{1,k_1,l_1}, \ldots, u^o_{n,1,l_n}, \ldots, u^o_{n,k_n,l_n}, u^s_{1,1,l_1}, \ldots, u^s_{1,k_1,l_1}, \ldots, u^s_{m,k_1,l_1}, \ldots, u^s_{m,k_m,l_m}\right\}$. Further, all u^a_{r,k_r,l_r} are real numbers. Their generation for each part of chromosome is described as follows: for the outsourcer's part, we randomly generate a real number between 0 and 1; this encoding is interpreted as follows: if we multiply this number by the maximum production rate, we obtain the percentage of maximal production rate to be adopted for in-house production plan in period k_i. For the subcontractors, as for outsourcers, we randomly generate a real number between 0 and 1; after multiplying this number by the maximum production rate, we obtain the additional percentage of maximal production rate to be allocated for the outsourcing for secondary outsourcers.

The choice of this type of encoding aims to avoid corrections after genetic operators. For the subcontractors, the strategic outsourcer's satisfaction is ensured by the strategic outsourcing plan, i.e. by the heuristic that computes the minimal production rate, but the secondary outsourcers are satisfied by the additional outsourcing plan, i.e. the additional percentage of maximal production rate and the rest of the strategic outsourcing plan. Figure 4 illustrates the structure of chromosome (Fig. 5).

Mechanism for generation of the initial population

All individuals initially generated or obtained after genetic operations require checking feasibility. The feasibility conditions ensures that the profit generated by each outsourcer i with outsourcing option is greater than or equal than the profit generated by this outsourcer without outsourcing. In addition, ensures that each outsourcer i does not accept terms of outsourcing contract with subcontractor j, except if the selling price proposed by this later is smaller or equal than the shortage cost generated if the demand is not meet by the outsourcer.

Fitness function

for the considered problem, we aim to optimize the total net profit of the whole system

Genetic operators

Selection: the adopted method for the selection of individuals to be used as parents to generate offspring for the next generation is the roulette method. This technique tends to promote the population diversity to avoid a premature convergence of the GA (Liu et al. 2008).

Crossover: two points crossover is implemented, based on a randomly generating cut points. For any two parents, their offspring are obtained by permutation of the different parts of the chromosome, as soon as the crossover operation is completed, the genes of the two chromosomes present within the two crossover points get interchanged. The genes before the first crossover point and the genes beyond the second crossover point remain unaltered even after the crossover operation.

Mutation: the mutation allows to the GA to reach all parts of the state space, without traveling all in the resolution process. For our procedure, we adopted the mutation by permutation,

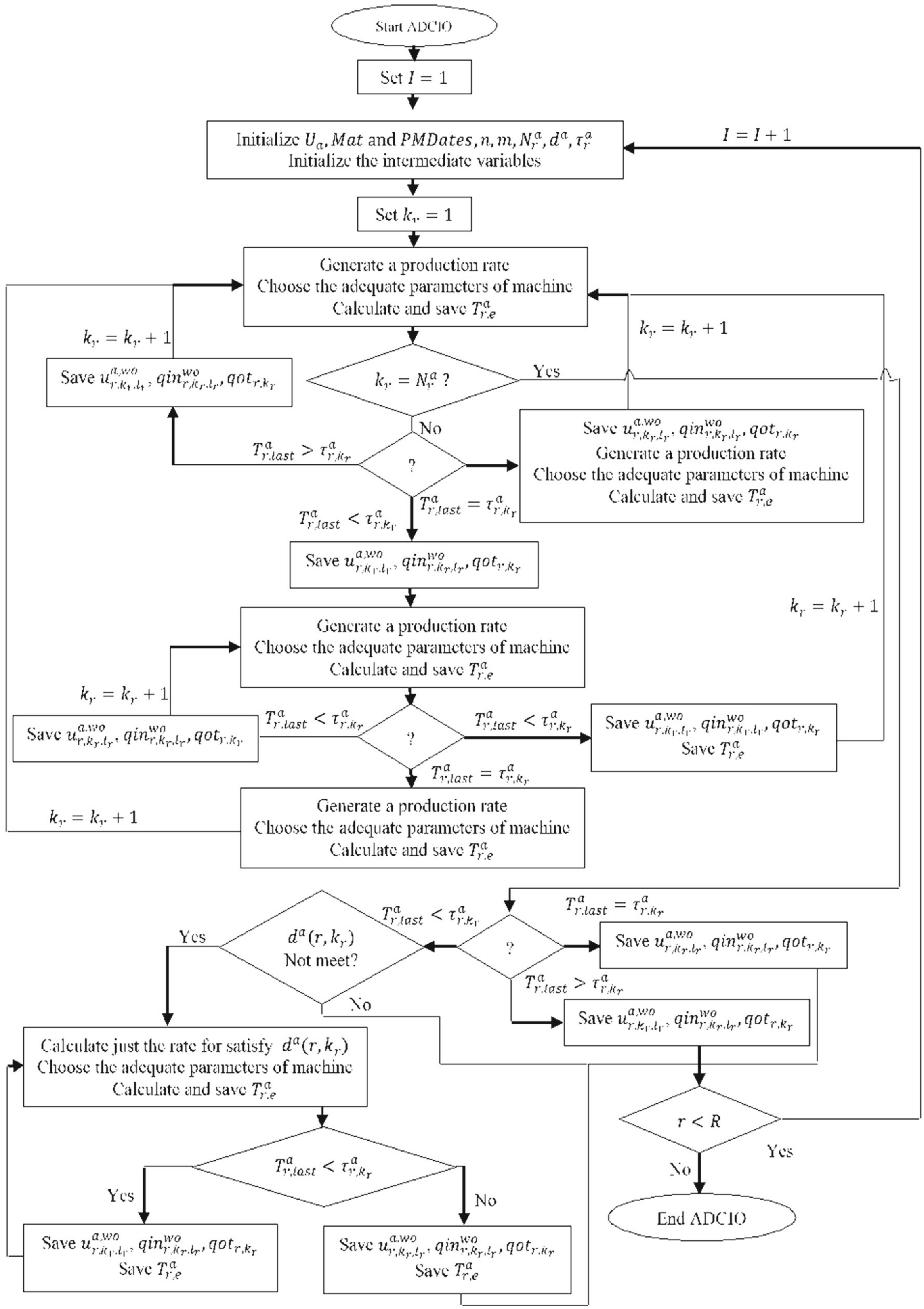

Fig. 5 Flow chart of ADCIO

where two positions are randomly selected and the genes they contain are permuted.

Replacement: this mechanism allows recombination between the new individuals created and those currently found in the population. In our procedure, the replacement phase was addressed in a hybrid manner, i.e. elitist and random replacement on both adopted.

Sizing parameters

The parameters of the genetic algorithm are set based on the experiments. The crossover rate, mutation rate, number of elites and random replacement are set to 0.65, 0.05, 0.15 and 0.15, respectively. The stopping criterion is based on the maximum number of generations and stall generations. In the experiments, each problem was run for 20 replications. The population size was set to 80. The number of generations is fixed at 150 and stall generations at 50.

Stopping criteria

The algorithm terminates if one or both of the following criteria are meet: the number of generations exceeds a predefined maximum number, and the best solution is not improved during a predefined number of generations.

We implement an algorithm to compute the optimal non-cyclic preventive maintenance date and all related costs. The algorithm is stated as follows:

Algorithm.1 Obtaining the Optimal Preventive Maintenance Date (2OPMD)

Initialization of different parameters: $\alpha_r^a, \beta_{r,l_r}^a, u_{r,k_r,l_r}^a, umax_r^a, t_s, t_e, pmc_{r,k_r,l_r}^a, cmc_{r,k_r,l_r}^a$

Main Program:

Find the minimum of the function $f_{MC}(T)$: the optimization of the maintenance cost is based on following result:

$\frac{\partial f_{MC}(T)}{\partial T} = 0$, where $f_{MC}(T) = \frac{CMC_r^a + PMC_r^a}{T}$

End.

The Heuristic for Obtaining the Weighted Minimum Production Rate of Company r (HOWMPRC) is shown as follows:

Heuristic.1 HOWMPRC

Initialization

$NbrProdRatePerPeriod$	Vector containing the number of production rates adopted during each period k_r;
$IndProdRatePeriod$	Vector containing the beginning and ending instants of sub periods corresponding to production rates used during each period k_r;
$ProdRate$	Dynamic vector containing the production rates during each period k_r;
$ProdSum$	Variable containing the cumulated production quantity;
$WeightedProdRate$	Vector containing the weighted production rates adopted during each period k_r;

$jj = 2$;

Main Program:

For $k_r = 1$ until N_r^a

$ProdSum = 0$;

While $j \leq NbrProdRatePerPeriod(k_r)$

$ProdSum = ProdSum + \big(IndProdRatePeriod(jj) - IndProdRatePeriod(jj-1)\big) * ProdRate(j)$;

$jj = jj + 1$;

$j = j + 1$;

End While

$WeightedProdRate(k_r) = \frac{ProdSum}{(IndProdRatePeriod(jj-1) - IndProdRatePeriod(jj - NbrProdRatePerPeriod(k_r)+1))}$;

End For

In the heuristic "computation of the matrix of allocation", the outsourcers accept or reject the outsourcing plans proposed by the platform after a comparative study between in-house production with shortage costs and mixed plans of in-house production and outsourcing. Only plans of outsourcing that are better than the in-house production plans with shortages are allowed. The heuristic for computation of the allocation matrix should respect three rules of allocation:

Rule 1: In order to minimize the storage cost of subcontractors, each one must meet the outsourcer who has the nearest delivery time.

Rule 2: If two outsourcers have the same delivery time, the subcontractor makes a fair distribution, i.e. proportional to the outsourcing amount issued.

Rule 3: In order to minimize the storage cost of outsourcers, each one must confide to the subcontractor where he not pays additional storage costs.

The heuristic is shown as follows:

Heuristic.2 Allocation Heuristic (AH)

Initialization:

Initialize $n, m, N_i^o, N_j^s, \overline{sp}_j^{s,sc}, sc_{ind}^o$;
The dynamic list containing the index of outsourcer ind;
The number of outsourcers those must satisfy in same time N_{idem};
The dynamic list containing *index of period in* $DelivTimeVect$ (S_{min});
The vector of delivering time of all outsourcers $DelivTimeVect$;
The remaining amount of the allocation to strategic outsourcers Q_r;
The outsourcing request of outsourcers Q_s;

Main Program:

$Limrepeat = \sum_{j=1}^{m} N_j^s$;
While $irepeat \leq Limrepeat$
$Min = +\infty$

Find an outsourcer who has a need for outsourcing and nearest delivery time

$S_{min} = [\,]$; $N_{idem} = 0$; $Limk = \sum_{i=1}^{n} N_i^o$;
For k_i*=1 until Limk*
If $DelivTimeVect(k_i) \leq Min$
If $DelivTimeVect(k_i) < Min$
$Min = DelivTimeVect(k_i)$;
Save the index k_i *in the list* S_{min} ;
Save the index of outsourcer ind;
$N_{idem} = N_{idem} + 1$;
Else
Save the index k_i *in the list* S_{min} ;
Save the outsourcers index ind;
$N_{idem} = N_{idem} + 1$;
End if
End if
End for

Allocate the remaining amount of the allocation to strategic outsourcers to secondary outsourcers.

If $N_{idem} = 1$
For j=1 until m
If $\overline{sp}_i^{s,sc} \leq sc_{ind}^o$
For each period k_j *of subcontractor j*
If $Q_r(j, k_j) \neq 0$ & $Min \leq \tau_{j,k_j}^s$ & $Q_s(ind) \neq 0$
Allocate the amount to the outsourcer ind and update Q_r *and* Q_s;
Set $DelivTimeVect(k_j) = H^s + 1$;
End if
End for
End if
End for
Else

```
For j=1 until m
  Remove from the list ind the outsourcers who refuse the outsourcing contract
  with the subcontractor j;
    For each period k_j of subcontractor j
      If Q_r(j, k_j) ≠ 0 & Min ≤ τ^s_{j,k_j} & Q_s(ind) ≠ 0
        Make a fair allocation of the amount between outsourcers of updated
        list ind and update Q_r and Q_s;
        Set DelivTimeVect(k_j) = H^s + 1;
      End if
    End for
  End for
  End if
irepeat = irepeat + 1;
End while
```

The following flow chart (Algorithm for Decoding of Chromosomes in Integrated Optimization "ADCIO") shows how we use the different heuristics and algorithms for decoding our chromosomes. For the outsourcer's part, we generate a production rate based on decoding of chromosome. However, at the subcontractor's part, we generate a production rate based on the heuristic HOWMPRC and decoding of chromosome.

Experimental design and computational results' analysis

In this section, we provide and discuss a numerical study that we performed by solving the formulation given in the "Mathematical models formulation". To show the robustness of proposed approach, an experimental design is developed, based on some critical planning parameters that to have a significant impact on solutions. The sensitivity analysis considers three key parameters: number of partners, costs structures and demands structures. We are primarily interested in the effects of number of outsourcers and subcontractors, different shortage and production costs levels and demand variation of subcontractors on adhesion percentage to the platform, realized profits and costs reduction, i.e. on the optimal decisions and the global system performances.

Problem data

The experimental design is structured and inspired from Aghezzaf et al. (2007), Aghezzaf and Najid (2008), Najid and Alaoui-selsouli (2011), Fitouhi and Nourelfath (2012). The problem dimensions are represented by the number of partners, respectively n and m, the number of levels in shortage and production costs problems, and finally the demand fluctuation. We use nine different combinations for the outsourcers and subcontractors:

$(n, m) \in \{((5,5);(5,10);(5,15);(10,5);(10,10);(10,15);(15,5);(15,10);(15,15)\}$. Maximum production rate is the first key parameter set for each production unit in each test problem. Because, it is the base for other parameters generation, such as the adopted production rate and its associate production cost, the inventory holding cost and the demands.

For each production unit of outsourcer, we fix $umax_i^o$ for three groups of test problems. $umax_i^o$ is selected randomly from $[40, 75]$. Failure distribution of each outsourcer's production unit is selected from Weibull distributions with shape parameters was set to 3 for all problems (low, medium, high and over high cost problems). The scale parameters are fixed at 17 for low problems, 16 for medium problems, 15 for high problems and 14 for over high problems. The maximal production rate of each machine of both outsourcers and subcontractors are given in Table 1.

The cost structure is supposed to get the following estates: the shortage cost is defined as $sc_i^o = \overline{sp}_i^o(1+\theta\%)$. For each outsourcer company θ is selected randomly from three different intervals $[15, 18]$ for low shortage cost problems, $[25, 28]$ for medium shortage cost problems and $[35, 38]$ for high shortage cost problems. The production cost pc^o_{i,k_i} depends on adopted production rate during each period k_i; this rate is randomly generated by the genetic algorithm meta-heuristic; production cost schedule is the parameter introduced to define the production rate/level in schedule (cadence tightness).The four levels correspond respectively to situations with loose, moderately loose, tight and over tight production rate. where:

$$\text{if } u_i^o \in \begin{cases}]0, 0.5umax_i^o] & \text{loose} \\ ]0.5umax_i^o, 0.75umax_i^o] & \text{moderately loose} \\ ]0.75umax_i^o, 0.9umax_i^o] & \text{tight} \\ ]0.9umax_i^o, umax_i^o] & \text{overtight} \end{cases}$$

Table 1 Maximal production rates

Machine (r)	1	2	3	4	5
$umax_r^o$	40	64	74	57	49
$umax_r^s$	67	114	51	91	73

Table 2 Periodic demands of outsourcers

Period	1	2	3	4	5
1	{3154,79}	{615,11}	{4393,46}	{1861,24}	{1962,31}
2	{763,99}	{1326,29}	{504,52}	{874,40}	{1002,49}
3	{473,110}	{1768,61}	{2557,86}	{2221,70}	{196,53}
4	{439,120}	{409,68}	{137,88}	{961,82}	{495,61}
6	{1746,155}	{3105,109}	{99,89}	{1426,102}	{2698,119}
8	{932,180}	{2669,147}	{3790,126}	{958,117}	{736,130}
10	–	{1990,175}	{1419,140}	{1545,136}	{2406,180}
12	–	{396,180}	{1162,153}	{427,143}	–
14	–	–	{543,161}	{2278,180}	–
15	–	–	{874,174}	–	–
16	–	–	{357,179}	–	–
17	–	–	{65,180}	–	–

Thus, pc_i^o is selected randomly from the above four situations.

$$\text{where } pc_i^o \in \begin{cases} [2, 3.5[& \text{if loose} \\ [3.75, 4.5[& \text{if moderately loose} \\ [4.75, 6.75[& \text{if tight} \\ [7, 8[& \text{if overtight} \end{cases}$$

The costs of PM actions are generated randomly as follows: from the intervals [10, 14], [16, 20], [22, 26] and [28, 32] respectively for low, medium, high, and over high PM cost problems. The costs of minimal repair actions at failures are generated randomly from the intervals [18, 23], [25, 30], [32, 37] and [39, 44], respectively, for low, medium, high and over high minimal repair cost problems.

As defined in Aghezzaf and Najid (2008), the holding cost is then written as $hc_{i,k_i}^o = \rho_o\% max\left(pc_{i,k_i}^o\right)$, where ρ_o is uniformly distributed between 5 and 20.

$$\text{where } pc_{i,k_i}^o = \begin{cases} \overline{pc}_{i,k_i}^o & \text{if} \exists\, T_{i,e}^o \in \left[\tau_{k_i-1}, \tau_{k_i}\right] \\ pc_{i,k_i}^o & \text{otherwise} \end{cases}$$

For all outsourcers and subcontractors, the planning horizon is set at 180 days. The demands data of each outsourcer and subcontractors are created as follows: the number of periods is generated randomly from the interval [6, 12] . The delivering times are generated randomly from the interval [1, 180] . The demands of outsourcers are uniformly distributed between 1 and $\Delta\tau_{k_i} umax_i^o\,(1+\delta_i\%)$. However, the demands of subcontractors are uniformly distributed between 1 and $\Delta\tau_{k_j} umax_j^s\left(1-\delta_j\%\right)$, where δ_r is uniformly distributed between 25 and 50. For each outsourcer, the periodic demands are presented in Table 2. Tables 3, 4 and 5 present three levels of demand fluctuation for each subcontractor. Finally, the profit rates for outsourcers are uniformly distributed between [35, 40] and randomly sampled from the set {15, 20} for the subcontractors. The initial stock of each outsourcer is randomly sampled from the set {0, 10} and {0, 20} for the subcontractors. The outsourcing cost of initial stock is randomly generated between [50, 90] .

As for the outsourcers, the maximum production rate and the failure distribution for each production unit of subcontractors are fixed for three groups of test problems. $umax_j^s$ is selected randomly from [50, 120]. In each period, the production rate is randomly generated by the genetic algorithm meta-heuristic. These values are averaged over all periods for the planning horizon. As mentioned previously, the average production rate is the base for other parameters generation. We consider four levels of production cost schedule; each level corresponds respectively to situations with loose, moderately loose, tight, and over tight production rate, where:

$$\text{if } u_j^s \in \begin{cases}]0, 0.5umax_j^s] & \text{loose} \\ ]0.5umax_j^s, 0.75umax_j^s] & \text{moderately loose} \\ ]0.75umax_j^s, 0.9umax_j^s] & \text{tight} \\ ]0.9umax_j^s, umax_j^s] & \text{overtight} \end{cases}$$

Thus, pc_j^s is selected randomly from the above four situations, where:

$$pc_j^s \in \begin{cases}]0.25, 0.75[& \text{if loose} \\ ]0.8, 1[& \text{if moderately loose} \\ ]1.1, 1.6[& \text{if tight} \\ ]1.8, 2.6[& \text{if overtight} \end{cases}$$

These selected unitary production costs are classified in four different categories [0.25, 0.75[, [0.8, 1[, [1.1, 1.6[and [1.8, 2.6[, respectively for low, medium, high and over high production cost problems.

Based on the explanations of outsourcers' side, the reliability and maintenance parameters are fixed as follows: the costs of preventive maintenance actions are generated randomly from the intervals [10, 14], [16, 20], [22, 26] and [28, 32], respectively for low, medium, high and over high PM cost problems. The costs of minimal repair actions at failures are generated randomly from the intervals [18, 23], [25, 30], [32, 37] and [39, 44], respectively, for low, medium,

Table 3 Periodic demands of subcontractors–Higher demands

Period	1	2	3	4	5
1	{4519,76}	{87,1}	{1278,29}	{1036,13}	{135,2}
2	{421,84}	{5537,63}	{775,45}	{1970,40}	{141,4}
3	{2777,136}	{4057,103}	{389,54}	{921,52}	{2650,42}
4	{280,141}	{5887,169}	{230,59}	{3983,107}	{197,45}
6	{2056,173}	{765,176}	{120,62}	{139,109}	{1906,75}
8	{386,180}	{363,180}	{39,63}	{1844,136}	{1399,100}
10	–	–	{1784,100}	{165,138}	{1309,119}
12	–	–	{1008,120}	{981,152}	{1262,138}
14	–	–	{114,123}	{839,162}	{197,141}
15	–	–	{781,142}	{454,167}	{1011,156}
16	–	–	{15,180}	{1021,180}	{1626,180}

Table 4 Periodic demands of subcontractors—Medium demands

Period	1	2	3	4	5
1	{2697,76}	{61,1}	{762,29}	{645,13}	{77,2}
2	{276,84}	{3830,63}	{438,45}	{1301,40}	{79,4}
3	{1806,136}	{2287,103}	{240,54}	{579,52}	{1451,42}
4	{184,141}	{3904,169}	{135,59}	{2632,107}	{113,45}
6	{1132,173}	{429,176}	{81,62}	{94,109}	{1185,75}
8	{245,180}	{240,180}	{26,63}	{1234,136}	{987,100}
10	–	–	{985,100}	{93,138}	{714,119}
12	–	–	{548,120}	{675,152}	{754,138}
14	–	–	{82,123}	{499,162}	{109,141}
15	–	–	{233,142}	{197,167}	{588,156}
16	–	–	{615,180}	{307,180}	{915,180}

Table 5 Periodic demands of subcontractors—Lower demands

Period	1	2	3	4	5
1	{2002,76}	{61,1}	{762,29}	{621,13}	{19,2}
2	{135,84}	{3830,63}	{438,45}	{1301,40}	{61,4}
3	{711,136}	{2287,103}	{240,54}	{579,52}	{1406,42}
4	{184,141}	{3904,169}	{135,59}	{2051,107}	{71,45}
6	{1132,173}	{95,176}	{81,62}	{67,109}	{1185,75}
8	{3,180}	{237,180}	{3,63}	{557,136}	{809,100}
10	–	–	{826,100}	{81,138}	{604,119}
12	–	–	{327,120}	{339,152}	{754,138}
14	–	–	{20,123}	{418,162}	{109,141}
15	–	–	{130,142}	{197,167}	{588,156}
16	–	–	{615,180}	{307,180}	{915,180}

high and over high MR cost problems. Failure distribution of each subcontractor's production unit is selected from Weibull distributions with shape parameters was set to 3 for all problems. The scale parameters are fixed at 17 for low problems, 16 for medium problems, 15 for high problems and 14 for over high problems. As defined in the outsourcers side, the holding cost of subcontractors is given by $hc^s_{j,k_j} = \rho_s\%max\left(pc^s_{j,k_j}\right)$, where ρ_s is uniformly distributed between 4 and 12.

It is useless to optimize each part separately, optimization should take into account both parts of the supply chain. In this subsection, we demonstrate the effectiveness of our integrated optimization approach.

Computational results and discussions

In this subsection we discuss the results of computational experiments carried out in the previous subsection. The tests aim to compare our integrated model with a separate model (non-integrated). We show that integrating in-house production and outsourcing increases the total profit for both outsourcers and subcontractors. The value of the study of mixed in-house production and outsourcing under integrated maintenance constraints will be then illustrated.

We used the language and interactive environment MATLAB® to code the instances generation and algorithms on a Pentium 4 with Intel Core 2 Duo CPU (2.93 GHz) processor and 2 GB RAM.

To optimize the performance of supply chain, managers use indicators and decision variables. Beamon (1998) developed a non-exhaustive list of indicators and decision variables typically used in management of the supply chain. In our analysis, we have used the number of actors that includes the chain; the percentage of actors those adhere to the platform, production planning (i.e. amount to produce or deliver), inventory levels, and maintenance periodicities... The experiment design realization is divided into three parts.

Table 6 Summary solution with lower and higher shortage costs

n	m	π_T^{sc}	Shortage cost			
			Low		High	
			Best solution	$\overline{te}$	Best solution	$\overline{te}$
5	5	SP	723242.44	0.0045	743059.58	0.0044
		IP	1171942.12	40.18	1994420.16	40.59
5	10	SP	1157008.91	0.0080	1190781.46	0.0065
		IP	1643186.01	62.76	1820840.45	65.57
5	15	SP	1113785.00	0.0120	1145116.40	0.0114
		IP	9172333.70	61.91	7057658.70	63.42
10	5	SP	1600168.53	0.0076	1633790.16	0.0063
		IP	3360067.65	79.18	3577950.71	79.64
10	10	SP	2185909.69	0.0092	2250898.68	0.0106
		IP	2779519.13	85.65	2988817.84	88.64
10	15	SP	2044358.46	0.0092	2094003.73	0.094
		IP	2953329.28	112.14	3196442.06	111.80
15	5	SP	2201441.31	0.0095	2245788.11	0.0098
		IP	3029536.05	101.67	34806698.79	99.67
15	10	SP	2371929.34	0.0112	2431586.96	0 .0015
		IP	2856302.71	139.16	3074381.55	145.58
15	15	SP	2895273.18	0.0120	2974964.81	0.0142
		IP	3600852.34	160.34	3889671.18	173.65

Variation of the number of outsourcers and subcontractors

For the first part, 9 trials are performed where each trial is obtained by the variation of the number of subcontractors and outsourcers. Note that all instances are solved in separate (SP) and integrated problems (IP). The corresponding optimal solutions, those are the total profit of supply chain and the average time of execution of all trials are given in Table 6.

The results show that the number of partners does' not affect the total profit of supply chain. It is the costs structures that determine the degree of partnership. For this, almost all problems dealing with the issue of supply chain management are elaborated upon the costs reduction in different levels of supply chain. The total costs reduction reduces the price of products and so seeks to acquire additional market share and generates profits for future investments. For example, in integrated approach with lower or higher shortage costs, the case $(n, m) \in (10, 5)$ realize a total profit greater than the profits realized by the cases $\{(10, 10)\,; (10, 15)\}$. From Table 6, let us note that our proposed approach obtain a reasonable computational time to solve the integrated problem. The computational time is varying from 0.0044 seconds to 0.0142 seconds for separated problems and varying from 40 seconds to 145 seconds for integrated problems. This is done just to get an idea on the computational time of the different instances.

Production cost influence

For the second part of the experiment design, we take the instance (5, 5). Three trials are realized where; production cost is low, medium and high. For each trial, two levels of shortage cost are considered. In order to evaluate the obtained results, for each trial we define for the outsourcers the percentage of cost reduction and for the subcontractors the percentage of cost increase. The percentage of additional generated profit of both outsourcers and subcontractors is also defined. This is due to the integration of in-house production and outsourcing strategy for the outsourcers, and the strategy of secondary outsourcing contract for the subcontractors.

Tables 7, 8 and 9 show the percentage of generated profit of both parts of supply chain (individual and whole part), the percentage of cost reduction with the negative sign, and the percentage of cost increase with the positive sign for lower, medium and higher production costs. The last row indicates the percentage of actors they increase their profits and reduce or increase their costs.

We observe that, if production costs are lower, the outsourcer prefers in-house production than of outsource outside. The profit realized by outsourcers increased by 44.34% compared with separate approach. The subcontractors realised an increase in profit of 149.42 %. The total profit of supply chain increases by 49.86 %. If we switch in case where production costs are medium, the percentages of

Table 7 Percentage of generated profit, cost reduction or increase—Lower production cost of outsourcers

r	l	$\pi_r^{o,wo}$	π_T^o	Outsourcer				Subcontractor					
				$\overline{pc}$	$\overline{hc}$	$\overline{mc}$	$\overline{sc}$	$\pi_r^{s,wo}$	π_T^s	$\overline{pc}$	$\overline{mc}$	$\overline{hc}$	π_T^{sc}
1	L	52.93	44.34	−3.53	−1.11	−31.23	81.20	6.37	149.42	107.99	25.99	292.72	49.86
	H	18.51		−35.02	−32.64	−39.34	−1550.57	0		0	0	0	
2	L	58.15		−12.03	−1.40	−87.72	23.47	137.73		95.66	65.24	142.61	
	H	65.61		−8.29	−5.56	14.27	−185.81	0		0	0	0	
3	L	27.02		−7.74	−8.56	−48.16	98.69	411.54		96.41	38.91	461.73	
	H	17.88		−30.41	−73.40	4.85	−32.81	0		0	0	0	
4	L	41.41		−77.04	−41.25	7.32	81.89	271.36		135.46	168.61	335.82	
	H	69.49		−24.11	−21.35	3.98	−33.48	666.18		138.58	172.79	314.15	
5	L	51.76		−3.28	−6.88	−39.57	−74.64	364.92		50.03	−0.59	242.52	
	H	64.02		−17.12	−6.16	−46.06	−177.44	55.15		64.29	145.33	314.91	
		100		100	100	60	60	70		70	60	70	

Table 8 Percentage of generated profit, cost reduction or increase—Medium production cost of outsourcers

r	l	$\pi_r^{o,wo}$	π_T^o	Outsourcer				Subcontractor					
				$\overline{pc}$	$\overline{hc}$	$\overline{mc}$	$\overline{sc}$	$\pi_r^{s,wo}$	π_T^s	$\overline{pc}$	$\overline{mc}$	$\overline{hc}$	π_T^{sc}
1	L	71.15	63.83	−7.54	−12.79	−2.68	93.21	372.47	147.99	105.20	−2.31	200.85	68.28
	H	36.95		−66.06	−116.53	−70.53	−4244.04	48.27		269.93	123.06	501.34	
2	L	106.37		0	−0.61	0.20	30.47	61.56		289.25	62.17	388.63	
	H	46.89		−60.11	−71.44	−19.68	−848.62	61.56		280.03	85.06	310.97	
3	L	37.28		−16.55	2.28	−144.23	−103.84	96.66		322.09	140.83	1202.10	
	H	69.09		−13.30	1.68	−52.00	−115.88	96.66		331.54	38.97	1315.91	
4	L	2.15		−114.16	−104.41	−80.37	−308.67	112.51		510.76	137.23	1445.74	
	H	106.96		−11.50	−11.21	−2.01	4.10	37.79		377.24	172.86	742.25	
5	L	60.17		−10.08	−4.31	16.25	−26.22	234.13		202.00	31.48	884.84	
	H	122.63		0.47	−1.02	1.90	6.90	594.11		198.76	31.41	829.74	

increased profits are 63.83%, 147.99% and 68.28%, respectively for outsourcers, subcontractors and supply chain. Likewise if the costs are higher, the increasing are 92.09%, more of two times, and almost 100%, respectively for outsourcers, subcontractors and all supply chain.

If we analyse the degree of partnership, we observe in each case almost all the partners adhere to the platform (all outsourcers, for the subcontractors almost everybody 70%, 100% and 100%). However, there is a significant divergence in the percentages of increased profits and the costs reduction percentages. This, upholds that he performance of supply chains improves by improving the overall performance of the chain, as a result, each company can improve its own performance (the reverse is not always true), but this assumes that the company is effectively coordinates with partners.

To illustrate the values of the study of mixed in-house production and outsourcing under integrated maintenance. We analyze the results of the experimental design presented in the Tables 7, 8 and 9. Us note that almost all outsourcers have reduced their costs by calling outsourcing services, except a reduced number that does' not reduce some costs. However, this later has not great influence on the profits. These costs reduction are due to the reduction in the in-house exploitation frequencies of machines. Similarly, it is obviously that the subcontractors does not engage in an agreement unless it is profitable. Consequently, it uses the non-integrated plan; its percentages of decreased costs are nulls. The costs of the rest of subcontractors are increased, due to the production at additional rate; it is taken into account during the costs computation, and consequently, the selling price.

Outsourcers' shortage cost influence

In all cases, when outsourcers' shortage cost is smaller than the selling price of subcontractors for secondary outsourcers, the integrated problem provides the same results as the separate problem. In the contrary case, the integrated problem gives better results (Table 6).

Table 9 Percentage of generated profit, cost reduction or increase—Higher production cost of outsourcers

r	l	$\pi_r^{o,wo}$	π_T^o	Outsourcer				Subcontractor					
				$\overline{pc}$	$\overline{hc}$	$\overline{mc}$	$\overline{sc}$	$\pi_r^{s,wo}$	π_T^s	$\overline{pc}$	$\overline{mc}$	$\overline{hc}$	π_T^{sc}
1	L	79.55	92.09	0.45	−0.05	0.46	−23.74	96.55	214.63	399.10	37.31	734.33	99.12
	H	119.21		0.56	0.61	−0.71	32.35	96.55		400.55	30.30	795.05	
2	L	78.53		−7.03	−6.98	−40.02	−103.00	524.94		302.70	12.94	278.87	
	H	110.71		−13.09	−15.82	−2.19	59.89	123.43		661.26	71.39	826.54	
3	L	77.77		−1.72	2.27	−36.74	20.37	65.43		368.31	141.28	1282.13	
	H	71.76		−13.32	8.65	−90.95	−72.21	543.36		238.81	71.59	564.26	
4	L	19.20		−39.36	−36.83	−117.70	17.19	73.73		671.12	36.75	1683.70	
	H	54.40		−24.29	−19.27	3.30	−51.72	73.73		814.57	139.33	2143.50	
5	L	44.95		−23.90	−12.63	11.14	40.85	74.49		411.72	137.97	1558.40	
	H	91.89		−0.48	2.59	−7.36	−369.51	272.56		348.33	0.23	1228.22	
		100		80	60	70	50	100		100	100	100	

Table 10 Percentage of generated profit, cost reduction or increase—Lower demand of subcontractors

r	l	$\pi_r^{o,wo}$	π_T^o	Outsourcer				Subcontractor					
				$\overline{pc}$	$\overline{hc}$	$\overline{mc}$	$\overline{sc}$	$\pi_r^{s,wo}$	π_T^s	$\overline{pc}$	$\overline{mc}$	$\overline{hc}$	π_T^{sc}
1	L	18.53	67.1	−25.22	−45.61	−35.53	−13.62	0	609.39	0	0	0	115.94
	H	5.18		−69.80	−41.36	−249.28	−159.15	0		0	0	0	
2	L	74.94		−11.48	−3.60	−91.44	−80.80	0		0	0	0	
	H	90.32		−9.67	−0.41	−124.50	−593.18	0		0	0	0	
3	L	69.62		−2.02	0.50	−34.75	16.43	0		0	0	0	
	H	9.35		−82.48	−57.54	−259.16	−186.84	0		0	0	0	
4	L	52.85		−3.43	−1.53	19.52	59.78	187.58		17.15	17.18	17.19	
	H	226.31		−6.52	−9.66	−42.28	−516.82	0		0	0	0	
5	L	63.79		−5.98	2.85	−1.48	−22.50	0		0	0	0	
	H	66.40		−16.26	−6.27	−40.81	91.57	80.21		4.85	5.62	16.49	
		100		100	80	90	70	20		20	20	20	

For the last part of the experiment design, three trials are realized where; three levels of demand fluctuation are considered.

Demand variation influence

The demand variation of subcontractors affects the total cost of both subcontractors and outsourcers. The adhesion to the outsourcing platform can obtain a better agreement in order to reduce the total cost of the outsourcers (when there is a subcontractor with a selling price for the secondary outsourcers equal or better than the shortage cost of outsourcers). In addition, it increases the total profit of subcontractors even in the case where costs increase due to the use of the system with maximal capacity. If we vary the demand of outsourcers and following the same reasoning of demand variation of subcontractors, the demand variation of outsourcers affects the total cost of both subcontractors and outsourcers.

In same logic of the second part of the experiment design, the following tables show the percentage of generated profit of both parts of supply chain, the percentage of cost reduction with the negative sign, and the percentage of cost increase with the positive sign for lower and higher fluctuation demand. The last row indicates the percentage of actors they increase their profits and reduce or increase their costs. It is obviously that the subcontractors they not improve its profits use the separate plan. For lower demand of subcontractors, we have obtained the results presented in the following tables:

From the Tables 10 and 11, we observe that, if the demands of strategic outsourcers addressed to subcontractors are lower compared with maximal capacity of each subcontractor. The subcontractors reserve the remaining capacity to the secondary outsourcers. They realized an increase in profit of more of six times of non-integrated approach. The outsourcers have increased their total profit by 67.1%; however, the supply chain profit is increased by 115.94%. Contrary, if the demands of strategic outsourcers are higher, the percentage of increased profit of supply chain decreased until 62.18% (which expresses almost half of the profit realized in the case with lower demands).

Table 11 Percentage of generated profit, cost reduction or increase—Higher demand of subcontractors

r	l	$\pi_r^{o,wo}$	π_T^o	Outsourcer				Subcontractor					
				$\overline{pc}$	$\overline{hc}$	$\overline{mc}$	$\overline{sc}$	$\pi_r^{s,wo}$	π_T^s	$\overline{pc}$	$\overline{mc}$	$\overline{hc}$	π_T^{sc}
1	L	52.96	77.03	−23.40	−9.57	−113.37	−1014.04	44.16	20.8	72.47	105.23	89.16	62.18
	H	51.82		−112.83	−187.73	−161.49	−9073.13	6234.65		28.22	40.08	47.51	
2	L	273.41		−30.20	−48.13	−77.81	−693.05	0		0	0	0	
	H	77.95		−26.04	−38.82	−44.88	−237.32	0		0	0	0	
3	L	3.54		−60.89	−99.06	−68.45	−172.10	96.25		22.11	12.29	75.92	
	H	64.72		−12.27	−20.31	9.99	9.74	0		0	0	0	
4	L	90.57		0	−0.67	2.42	33.44	1173.48		161.16	142.76	411.97	
	H	77.64		−23.61	−13.28	−26.14	−145.08	313.15		283.63	317.34	676.86	
5	L	48.68		−26.55	−16.68	7.40	−277.04	0		0	0	0	
	H	94.97		−8.84	−3.44	−77.30	−213.67	0		0	0	0	
		100		90	100	70	80	50		50	50	50	

Conclusions and future research directions

Through this paper, we have studied an outsourcing platform in supply chain environment according to the characteristics of the integrated maintenance planning. The studied system is a two-echelon supply chain consisting of multi-outsourcers and multi-subcontractors; each one has a failure-prone single-machine that produces a single product. The aim of this model is to develop a collaborative outsourcing manufacturing chain through bidirectional selection among them.

Given a specific configuration of outsourcing, we have developed optimization algorithms to find simultaneously the optimal integrated in-house production/outsourcing and maintenance plans for each outsourcer, and the integrated secondary/strategic outsourcing and maintenance plans for each subcontractor. The aim is to maximize the total profit of whole system over a specified finite planning horizon. The optimal solution includes the production rates, the non-cyclic PM periods, the appropriate outsourcing providers and the outsourcing quantities.

The problem is solved by comparing the results of several problems test. The value of the study of the outsourcing under combined approach of production control and maintenance management and that of using noncyclical preventive maintenance when the demand varies from one period to another are illustrated and validated by a design of experiment. This later has shown that the outsourcing optimization which takes into account the integration of maintenance and production planning can reduce the total costs and the removal of periodicity constraint is directly affected by the demand fluctuation and can also reduce the total maintenance and production cost.

In addition to in-house production in regular time and outsourcing, overtime is another solution that can be considered by comparing different options. Such decisions usually involve other more complicated issues that are not included in the current models.

It would be interesting as future work to address the outsourcing problem in the multi-product context.

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